Algorithms

1. Asymptotic Notations and Apriori Analysis

For Micro Notes by the Student

01. **State True / False**

1. $100n.\log n = O(n.\log n)$
2. $2^{n+1} = O(2^n)$
3. $2^{2n} = O(2^n)$
4. $0<x<y$ then $n^x = O(n^y)$
5. $(n+k)^m \neq \theta(n^m)$ $(k, m) > 0$
6. $\sqrt{\log n} = O(\log \log n)$
7. $\log n$ is Ω $(1/n)$
8. 2^{n^2} is $O(n!)$
9. n^2 is $O(2^{2\log n})$
10. $a^n \neq O(n^x)$, $a >1$, $x > 0$
11. $2^{\log_2 n^2}$ is $O(n^2)$

02. **Asymptotic Comparisons**

01. $f(n) = n$, $g(n) = \log n$

02. $f(n) = n^2\log n$, $g(n) = n.\log^{10} n$

03. $f(n) = n^3$, $0 < n \leq 10{,}000$

$= n$, $n > 10{,}000$

$g(n) = n$, $0 < n \leq 100$

$= n^3$, $n > 100$

03. Two Packages are available for processing a Data Base having 10^x records. Package 'A' takes a times of $10.n.\log n$ while package B takes a time of $0.0001n^2$ for processing B records. Determine the smallest integer x for which Package 'A' outperforms Package 'B'.

04. **Arrange the functions in increasing order of rates if growth**

01. $n^2; n.\log n; n\sqrt{n}; e^n; n; 2^n; (1/n)$.

02. 2^n; $n^{3/2}$; nlogn; $n^{\log n}$

03. $n^{(1/3)}$; e^n; $n^{7/4}$; $n\log^9 n$; 1.001^n

05. Consider the following functions from positive integers to real numbers:

$10, \sqrt{n}, n, \log_2 n, \frac{100}{n}$.

The CORRECT arrangement of the above functions in increasing order of asymptotic complexity is:

(a) $\log_2 n, \frac{100}{n}, 10, \sqrt{n}, n$

(b) $\frac{100}{n}, 10, \log_2 n, \sqrt{n}, n$

(c) $10, \frac{100}{n}, \sqrt{n}, \log_2 n, n$

(d) $\frac{100}{n}, \log_2 n, 10, \sqrt{n}, n$

06. Which of the following is **TRUE**?

f(n) is O(g(n))
g(n) is NOT O(f(n))
g(n) is O(h(n))
h(n) is O(g(n))

(a) f(n) is O(h(n))
(b) f(n) + h(n) is O(g(n)+h(n))
(c) $h(n) \neq O(f(n))$
(d) $f(n) . g(n) \neq O(g(n)). h(n))$

07. An element in an Array is called Leader if it is greater than all elements to the right of it. The Time Complexity of the most efficient Algorithm to print all Leaders of the given Array of size 'n' is _____.

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08. Consider a Binary Tree where root is at level 1 and each other level 'i' of the binary Tree has exactly 'i' nodes. The height of such a binary Tree having 'n' nodes is order of ______.

09. Consider the following operation along with EnQueue & DeQueue operation on queues where 'k' is a global parameter.

```
MultiQueue (Q)
{
 m = k;
 while ((q is not empty) and (m > 0))
 {
     DeQueue (q);
     m = m – 1;
 }
}
```

What is the worst case time complexity of a sequence of 'n' Queue operations on an initially empty Queue?

(a) O(n) (b) O(n+k)
(c) O(n.k) (d) $O(n^2)$

10. A Queue is implemented using an Array such that EnQueue DeQueue operations are performed efficiently. Which one of the following statements is correct for a Queue of size 'n'.

(a) Both operations can be performed in O(1)
(b) Worst complexity of both operation will be $\Omega(n)$
(c) Worst case time of both operation will be $\Omega(\log n)$
(d) Atleast one operation can be performed in O(1) time and the worst case time for the option will be $\Omega(n)$.

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Time Complexity Framework for Recursive Algorithms

11. Algorithm What (n)
```
{
 if (n = 1) return;
 else
 {
     What (n – 1);
      B(n);
 }
}
```

12. Algorithm A(n)
```
{
        if (n = 2) return;
        else
        return (A(√n));
}
```

13. Algorithm Do_It (n)
```
{
        if (n = 1) return;
        else
        return (Do_It (n – 1) + Do_It (n – 1));
}
```

14. Algorithm *A*(n)
```
{
        if (n = 2) return;
        else
        return (A(√n) + A(√n));
}
```

15. Algorithm Recur (n)

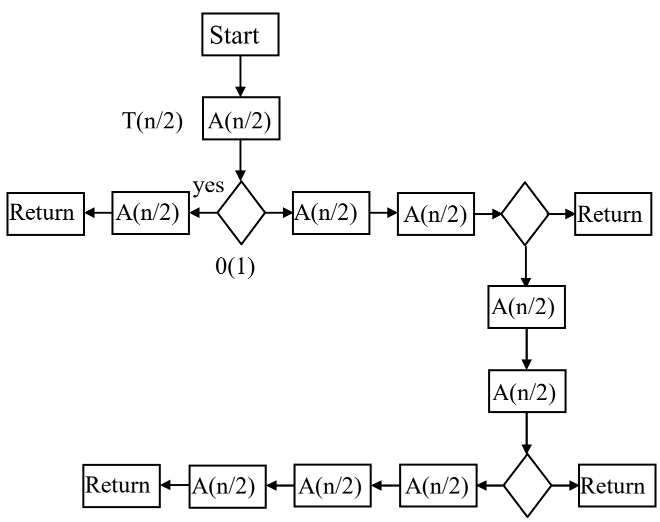
```
{
    if (n = 1) return;
    else
    {
        recur(n/2);
        recur(n/2);
        B(n);
    }
}
```

16. T(n) = 2, n = 2

$$= \sqrt{n}.T(\sqrt{n}) + (n), \quad n > 2$$

$T(n) = 2 \quad , \quad n = 2$

17. The given diagram represents the flowchart of recursive algorithm A(n). Assume that all statements except for the recursive calls have order(1) time complexity. Then the best case & worst case time of this algorithm is ________.



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18.
```
void abc(char *s)
{
 if(*s ! = '\0')
 {
    printf("%c",*s);
    abc(s + 1);
    abc(s+1);
 }
}
```

Running Times of Program Segments with Loops:

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19.
```
for i ← 1 to n
    c = c + 1;
```

20.
```
for i ← 1 to n
    for j ← 1 to n/2
        c = c + 1;
```

21.
```
for i ← 1 to n
    for j ← 1 to n/4
        for k ← 1 to n
            break;
```

22.
```
for (i= 1; i < = n; + + i)
    for (j = 1; j < = n; + + j)
        for (k = n/2, k < = n; k + = n/2)
            c = c + 1;
```

23.
```
i = 1;
while (i < = n)
{
    i = i * 2;
}
```

24.
```
i = n;
while (i > 0)
{
    i = i/2;
}
```

25.
```
k = 1; i = 1;
while (k < = n)
{
      i + + ;
      k = k + i ;
}
```

26
```
for (i = 1; i < = n; ++i)
   for (j = 1; j < n; j = 2 * j)
      c + c + 1;
```

27.
```
m = 2^n
for (i = 1; i < = n; + + i)
   for (j = 1; j < = m; j = 2 * j)
      c = c + 1
```

28.
```
for i ← 1 to n
  for j ← i to n
```

29. $f(n) = \sum_{i=1} O(n)$

30.
```
i = n;
while (i > 0)
{
      j = 1;
      while (j < = n)
      {
            j = 2 * j;
      }
      i = i/2;
}
```

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31. int fun (int n)

```
{
        int i, j, p, q = 0 ;
        for (i = 1; i < = n ; + + i)
        {
              p = 0;
              for (j = n; j > 1; j = j/2)
                    + + p;
              for (k = 1; k < p; k = k * 2)
                    + + q;
        }
        return (q);
}
```

32. for (i = 1; i < = n; + + i)

```
{
        j = 1;
        while (j < = n)
              j = 2 * j;
        for (k = 1; k < = n; + + k)
              c = c + 1;
}
```

33. $n = 2^{2^k}$

```
for (i = 1; i < = n; + + i)
{
        j = 2;
        while (j < = n)
        {
              j = j * j;
              pf("*");
        }
}
```

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34. A: Array [1 ……n] of binary;

```
f(m) = θ (m)
count: integer;
count = 1;
for i = 1 to n
{
      if (A[i] = = 1) count + +;
      else
      {
            f(count);
            count = 1;
      }
}
```

35. Assume that Merge Sort takes 30sec to Sort 64 elements in worst case. What is the approximate number of elements that can be Sorted in the Worst Case using Merge Sort using 6 minutes?

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2. Greedy Method

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36. Consider the weights and values of items listed below. Note that there is only one unit of each item.

Item number	Weight (in Kgs)	Value (in Rupees)
1	10	60
2	7	28
3	4	20
4	2	24

The task is to pick a subset of these items such that their total weight is no more than 11 Kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt}. A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by V_{greedy}. The value of $V_{opt} - V_{greedy}$ is_______.

37. We are given 9 tasks $T_1, T_2, \ldots, T_9$. The execution of each task requires one unit of time. We can execute one task at a time. Each task T_1 has a profit P_i and a deadline d_i, Profit P_t is earned if the task is completed before the end of the Deadline.

Task	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9
Profit	15	20	30	18	18	10	23	16	25
Deadline	7	2	5	3	4	5	2	7	3

a. Are all tasks completed in the schedule that gives maximum profit?
(a) All tasks are completed
(b) T_1 and T_6 are left out
(c) T_1 and T_8 are left out
(d) T_4 and T_6 are left out

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b. What is the maximum profit earned?
(a) 147 (b) 165 (c) 167 (d) 175

38. The characters 'a' to 'h' have the set of frequencies based on the first 8 Fibonacci numbers as follows :

a:1, b:1, c:2, d:3, e:5, f:8, g:13, h:21
A Huffman code is used to represent the characters. What is the sequence of characters corresponding to the following code?

110111100111010

(a) fdheg (b) ecgdf
(c) dchfg (d) fehdg

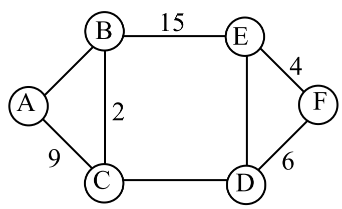
39. A message is made up entirely of characters from the set X = {P, Q, R, S, T}. The table of probabilities for each of the characters is shown below:

Character	Probability
P	0.22
Q	0.34
R	0.17
S	0.19
T	0.08
Total	1.00

If a message of 100 characters over X is encoded using Huffman coding, then the expected length of the encoded message in bits is _________.

40. Consider a Graph whose vertices are points in a plane with integer co-ordinates (x, y) where $1 \le x \le n$, $1 \le y \le n$, $n > 2$ is an integer. 2 vertices $<x_1, y_1>$ & $<x_2, y_2>$ are adjacent iff $|x_1 - x_2| \le 1$ & $|y_1 - y_2| \le 1$. The cost of such an edge is given by the distance between them. Compute the weight of min cost Spanning Tree of such graph for a value of n.

41. Consider the following Graph whose Minimum Cost Spanning Tree marked with edge values has a weight of 36. Minimum possible sum of all edges of the graph G is _____. (Assume that all edges have distinct cost).



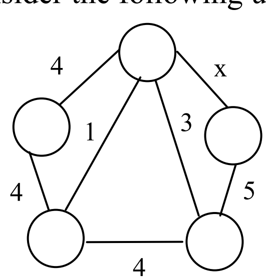
42. Consider a graph with 'n' vertices n > 2. The vertices are numbered V_1 to V_n. Two vertices V_i & V_j are adjacent iff $0 < |i - j| \leq 2$. The weight of such an edge is i + j. The weight of minimum cost Spanning Tree of such a graph for a value of n is _______.

43. Consider a complete weighted Graph with n vertices numbered V_1 to V_n. Two vertices V_i & V_j having edge between them has a cost value of $2|i - j|$. The weight of minimum cost Spanning Tree of such a graph is _______.

44. Let G be a complete undirected graph with 4 vertices and edge weights are {1, 2, 3, 4, 5, 6}. The maximum possible weight that a minimum weight Spanning Tree can have is _______.

45. Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes_______.

46. Consider the following undirected graph G:



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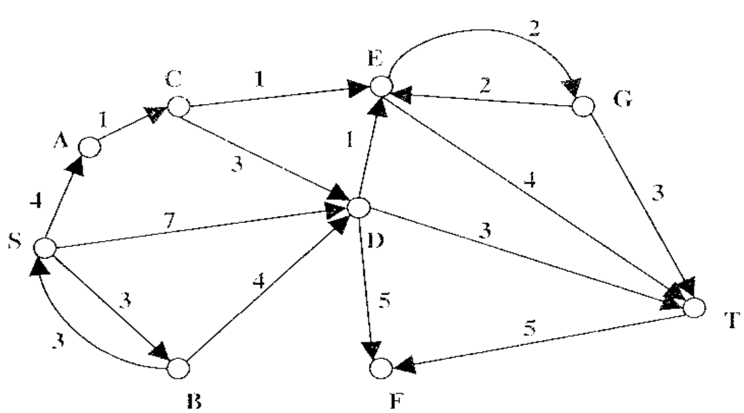
Choose a value for x that will maximize the number of minimum weight spanning trees (MWSTs) of G. The number of MWSTs of G for this value of x is_____.

47. Let w be the minimum weight among all edge weights in an undirected connected graph. Let 'e' be a specific edge of weight 'w'. Which of the following is False?
    i. There is a minimum Spanning Tree containing 'e' always.
    ii. Every minimum Spanning Tree has an edge of weight 'w'.
    iii. 'e' is present in every minimum Spanning Tree.
    iv. If 'e' is not present in a minimum Spanning Tree named 'T' then there will be a cycle formed by adding 'e' to T.

48. G = (V, E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are **TRUE?**
    I. If e is the lightest edge of some cycle in G, then every MST of G <u>includes</u> e
    II. If e is the heaviest edge of some cycle in G, then every MST of G <u>excludes</u> e
    (a) I only (b) II only
    (c) both I and II (d) neither I nor II

49. Applying Dijkstra's Algorithm over the given Graph, Which path is reported from 'S' to 'T';



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50. Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are true?
    1. Minimum spanning Tree of the graph does not change.
    2. Shortest path between any pair of vertices does not change.

51. Consider the weighted undirected graph with 4 vertices, where the weight of edge {i,j} is given by the entry W_{ij} in the matrix W.

$$W = \begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$$

The largest possible integer value of x, for which at least one shortest path between some pair of vertices will contain the edge with weight x is _____.

52. Let G = (V, E) be any connected undirected edge-weighted graph. The weights of the edges in E are positive and distinct. Consider the following statements:
    (I) Minimum Spanning Tree of G is always unique.
    (II) Shortest path between any two vertices of G is always unique.

    Which of the above statements is/are necessarily true?

    (a) (I) only (b) (II) only
    (c) both (I) and (II) (d) neither (I) nor (II)

3. Components

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53. A DFS is performed on DAG. Which of the following is true for all edges (u, v) in the graph?
(a) d[u] < d[v]
(b) d[u] < f[v]
(c) f[u] < f[v]
(d) f[u] > f[v]

54. Consider a DFT of an undirected graph having 'n' vertices. In the traversal, k edges are marked as Tree edges then the number of connected components in the graph is given by
(a) k
(b) k + 1
(c) n – k
(d) n – k – 1

4. Heap

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55. Valid binary Max-Heap
    (a) <25, 12, 16, 13, 10, 8, 14>   (b) <25, 14, 16, 13, 10, 8, 12>
    (c) <25, 14, 13, 16, 10, 8, 12>   (d) <25, 14, 12, 13, 10, 8, 16>

56. Valid 3-ary maximum Heap Array representation
    (a) <1, 3, 5, 6, 8, 9>   (b) <9, 6, 3, 1, 8, 5>
    (c) <9, 3, 6, 8, 5, 1>   (d) <9, 5, 6, 8, 3, 1>

57. To the valid Heap of Q55 insert elements < 7 2 10 4 > . Indicate the resultant Heap in Array.

58. Level order traversal of a binary max Heap generates: <10, 8, 5, 3, 2>
    Insert: <1 & 7> ; What is the resultant Level order Traversal?

59. In a binary max-Heap with n elements, the smallest element can be found in time of ______.

60. Given binary Heap with 'n' elements & it is required to insert 'n' more elements not necessarily one after another into this Heap. Total time required for this operation is:
    (a) $O(n^2)$ (b) nlogn   (c) n   (d) n^2logn

61. Given binary Heap in Array with the smallest at the root, the 7th smallest element can be found in time complexity of ______.

62. Consider binary Heap in an Array with n elements it is desired to insert an element into the Heap if a binary search is performed along the path from newly inserted element to the root then the no. of comparison made is order of _________.

63. The approximate no. of elements that can be Sorted in O(logn) time using Heap Sort is ________.

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64. Given $\lceil \log n \rceil$ Sorted lists each having $\lfloor n/\log n \rfloor$ elements. The time complexity to merge the given list into a single Sorted list, using Heap data structure is _____.

65. An operator delete(i) for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is implemented in an array and i refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?
(a) O(1) (b) O(d) but not O(1)
(c) $O(2^d)$ but not O(d) (d) $O(d2^d)$ but not $O(2^d)$

66. The minimum number of interchanges needed to convert the array into a max-heap is
89, 19, 40, 17, 12, 10, 2, 5, 7, 11, 6, 9, 70
(a) 0 (b) 1 (c) 2 (d) 3

67. An array of integers of size n can be converted into a heap by adjusting the heaps rooted at each internal node of the complete binary tree starting at the node $\lfloor (n-1)/2 \rfloor$ and doing this adjustment up to the root node(root node is at index 0) in the order $\lfloor (n-1)/2 \rfloor$, $\lfloor (n-3)/2 \rfloor$,...., 0. The time required to construct a heap in this manner is
(a) O(log n) (b) O(n)
(c) O(n log log n) (d) O(n log n)

68. An array X of n distinct integers is interpreted as a complete binary tree. The index of the first element of the array is 0. If only the root node does not satisfy the heap property, the algorithm to convert the complete binary tree into a heap has the best asymptotic time complexity of
(a) O(n) (b) O(log n)
(c) O(n log n) (d) O(n log log n)

69. Consider a complete binary tree where the left and right subtrees of the root are max-heaps. The lower bound for the number of operations to convert the tree to a heap is

(a) Ω (log n) (b) Ω (n)
(c) Ω (n log n) (d) Ω (n^2)

70. Consider a max heap, represented by the array:
40, 30, 20, 10, 15, 16, 17, 8, 4.

Array index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new hope is

(a) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35
(b) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
(c) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15
(d) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

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5. Sorting Techniques

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71. Which of the following Sorting algorithms has lowest worst case complexity?
    i. Bubble Sort ii. Merge Sort
    iii. Quick Sort iv. Selection Sort

72. Which of the following in place Sorting algorithm needs minimum number of swaps?
    (a) Selection Sort (b) Insertion Sort
    (c) Heap Sort (d) Quick Sort

73. What would be the worst case complexity of Insertion Sort if the inputs are restricted to permutation of 1 to n with at most 'n' Inversions?

74. Let 'S' be a Sorted Array of 'n' integers and T(n) denote the time taken for the most efficient algorithm to determine if there are 2 elements in the Array with the sum <1000.
    (a) T(n) is O(1) (b) $n \le T(n) \le n \log n$
    (c) $T(n) = {}^{n}C_2$ (d) $n\log n \le T(n) = {}^{n}C_2$

75. The traditional Insertion Sort to Sort an Array of n elements uses linear search to identify the position where an element is to be inserted into already Sorted part of the Array, if instead binary search is used to identify the position of newly inserted element then the worst case complexity will be order of ________.

76. In using Quick Sort suppose the central element of the Array is always chosen as the Pivot then the worst case complexity of the Quick Sort may be_____.

77. The Median on Array of size n can be found in O(n) time. If Median is selected as Pivot, then the worst case complexity of Quick Sort is ______.

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78. In applying Quick Sort to an unsorted list if (n/4)th Smallest element is selected as Pivot with a time complexity of O(n), then the Time Complexity of Quick Sort will be______.

79. Consider the Quicksort algorithm. Suppose there is a procedure for finding a pivot element which splits the list into two sub lists each of which contains at least one-fifth of the elements. Let T(n) be the number of comparisons required to sort n elements. Then
(a) $T(n) \leq 2T(n/5)+n$ (b) $T(n) \leq T(n/5)+T(4n/5)+n$
(c) $T(n) \leq 2T(4n/5)+n$ (d) $T(n) \leq 2T(n/2)+n$

80. Which one of the following in place sorting algorithms needs the minimum number of swaps?
(a) Quick sort (b) Insertion sort
(c) Selection sort (d) Heap sort

81. If one uses straight two-way merge sort algorithm to sort the following elements in ascending order:
20, 47, 15, 8, 9, 4, 40, 30, 12, 17
then the order of these elements after second pass of the algorithm is:
(a) 8, 9, 15, 20, 47, 4, 12, 17, 30, 40
(b) 8, 15, 20, 47, 4, 9, 30, 40, 12, 17
(c) 15, 20, 47, 4, 8, 9, 12, 30, 40, 17
(d) 4, 8, 9, 15, 20, 47, 12, 17, 30, 40

82. You have n lists, each consisting of m integers sorted in ascending order. Merging these lists into a single sorted list will take time:
(a) O(nm log m) (b) O(mn log m)
(c) O(m + n) (d) O(mn)

83. If we use Radix Sort to sort n integers in the range $(n^{k/2}, n^k]$, for some k > 0 which is independent of n, the time taken would be
(a) $\Theta(n)$ (b) $\Theta(kn)$
(c) $\Theta(n \log n)$ (d) $\Theta(n^2)$

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84. The worst case running times of Insertion sort, Merge sort and Quick sort, respectively are:
(a) $\Theta(n \log n)$, $\Theta(n \log n)$ and $\Theta(n^2)$
(b) $\Theta(n^2)$, $\Theta(n^2)$ and $\Theta(n \log n)$
(c) $\Theta(n^2)$, $\Theta(n \log n)$ and $\Theta(n \log n)$
(d) $\Theta(n^2)$, $\Theta(n \log n)$ and $\Theta(n^2)$

85. Let P be a quick sort program to sort numbers in ascending order.
Let t_1 and t_2 be the time taken by the program for the inputs [1 2 3 4] and [5 4 3 2 1], respectively. Which of the following holds?
(a) $t_1 = t_2$ (b) $t_1 > t$
(c) $t_1 < t_2$ (d) $t_1 = t_2 = 5 \log 5$

86. Let P be a Quick Sort Program to sort numbers in ascending order suing the first element as pivot. Let t1 and t2 be the number of comparisons made by P for the inputs {1, 2, 3, 4, 5} and {4, 1, 5, 3, 2} respectively.
Which one of the following holds?
(a) t1 = 5 (b) t1 < t2
(c) t1 > t2 (d) t1 = t2

87. Quick-sort is run on two inputs shown below to sort in ascending order taking first element as pivot
i. 1, 2, 3….n
ii. n, n−1, n −2,…,2,1
Let C_1 and C_2 be the number of comparisons made for the inputs (i) and (ii) respectively. Then,
(a) $C_1 < C_2$ (b) $C_1 > C_2$
(c) $C_1 = C_2$ (d) We cannot say anything for arbitrary n

88. Following algorithm(s) can be used to sort n in the range [1..n^3] in O(n) time
(a) Heap sort (b) Quick sort
(c) Merge sort (d) Radix sort

89. For merging two sorted lists of sizes m and n into a sorted list of size m+n, we require comparisons of
(a) O(m) (b) O(n)
(c) O(m+n) (d) O(log m+ log n)

90. Give the correct matching for the following pairs:

(A) O(log n)	(P) Selection sort
(B) O(n)	(Q) Insertion sort
(C) O(n log n)	(R) Binary search
(D) O(n^2)	(S) Merge sort

For Micro Notes by the Student